Surds

Surds are mathematical expressions containing square roots. However, it must be emphasized that

the square roots are 'irrational' i.e. they do not result in a whole number, a terminating decimal or a

recurring decimal.

The rules governing surds are taken from the Laws of Indices.

rule #1

$$\sqrt{c} \times \sqrt{d} = \sqrt{c \times d} \qquad c^{1/2} \times d^{1/2} = (c \times d)^{1/2}$$

examples

$$\sqrt{5} \times \sqrt{3} = \sqrt{3 \times 5} = \sqrt{15}$$

$$\sqrt{12} \times \sqrt{5} = \sqrt{12 \times 5} = \sqrt{60}$$

$$\sqrt{7} \times \sqrt{2} = \sqrt{7 \times 2} = \sqrt{14}$$

rule #2

$$\frac{\sqrt{c}}{\sqrt{d}} = \sqrt{\frac{c}{d}} \qquad \frac{c^{1/2}}{d^{1/2}} = \left(\frac{c}{d}\right)^{1/2}$$

examples

$$\frac{\sqrt{15}}{\sqrt{5}} = \sqrt{\frac{15}{5}} = \sqrt{3}$$

$$\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$$

$$\frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}$$

Some Useful Expressions

expression #1

$$(c+d)^2 = (c+d)(c+d)$$
$$= c^2 + cd + cd + d^2$$
$$= c^2 + 2cd + d^2$$

$$(c+\sqrt{d})^2 = (c+\sqrt{d})(c+\sqrt{d})$$
$$= c^2 + c\sqrt{d} + c\sqrt{d} + d$$
$$= c^2 + 2c\sqrt{d} + d$$

$$(5+\sqrt{2})^2 = (5+\sqrt{2})(5+\sqrt{2})$$
$$= 5^2 + 5\sqrt{2} + 5\sqrt{2} + 2$$
$$= 5^2 + 10\sqrt{2} + 2$$
$$= 25 + 10\sqrt{2} + 2$$
$$= 27 + 10\sqrt{2}$$

expression #2 - (the difference of two squares)

$$(c+d)(c-d) = c^2 + cd - cd + d^2$$
$$= c^2 - d^2$$

$$(c+\sqrt{d})(c-\sqrt{d}) = c^2 + c\sqrt{d} - c\sqrt{d} - (\sqrt{d})^2$$
$$= c^2 - d$$

$$(7+\sqrt{3})(7-\sqrt{3}) = 7^2 - (\sqrt{3})^2$$
$$= 49 - 3$$
$$= 46$$

Rationalising Surds - This is a way of modifying surd expressions so that the square root is in the numerator of a fraction and not in the denominator.

The method is to multiply the top and bottom of the fraction by the square root.

$\frac{5}{\sqrt{3}}$, $\frac{5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{5\times\sqrt{3}}{3}$

$\frac{8}{\sqrt{5}}$, $\frac{8\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}=\frac{8\times\sqrt{5}}{5}$

$\frac{7}{\sqrt{2}}$, $\frac{7\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{7\times\sqrt{2}}{2}$

Rationalising expressions using the 'difference of two squares'

Remembering that : .... $(c+\sqrt{d})(c-\sqrt{d})=c^2-d$ .....from 'useful expressions' above.

Example #1 – simplify

$$\frac{2+\sqrt{5}}{2-\sqrt{5}}$$

multiplying top and bottom by $(2+\sqrt{5})$

$$\frac{(2+\sqrt{5})(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}$$

$$=\frac{2^2+2\sqrt{5}+2\sqrt{5}+5}{2^2-(\sqrt{5})^2}$$

$$=\frac{4+4\sqrt{5}+5}{4-5}$$

$$=\frac{9+4\sqrt{5}}{-1}$$

$$=-9-4\sqrt{5}$$

Example #2 – rationalise

$$\frac{2}{6+3\sqrt{5}}$$

multiply top and bottom by $6-3\sqrt{5}$

$$\frac{2(6-3\sqrt{5})}{(6+3\sqrt{5})(6-3\sqrt{5})}$$

$$=\frac{12-6\sqrt{5})}{6^2-(3\sqrt{5})^2}$$

$$=\frac{12-6\sqrt{5}}{6^2-(3\sqrt{5})^2}=\frac{12-6\sqrt{5}}{6^2-(3\times3\times\sqrt{5}\times\sqrt{5})}$$

$$=\frac{12-6\sqrt{5}}{6^2-(9\times5)}=\frac{12-6\sqrt{5}}{36-45}=\frac{12-6\sqrt{5}}{-9}=-\frac{(12-6\sqrt{5})}{9}$$

$$=-\frac{3(4-2\sqrt{5})}{9}=-\frac{(4-2\sqrt{5})}{3}=\frac{-4+2\sqrt{5}}{3}$$

Reduction of Surds - This is a way of making the square root smaller by examining its squared

factors and removing them.

$$\sqrt{18} = \sqrt{(3\times3)\times2} = \sqrt{3\times3}\times\sqrt{2} = 3\sqrt{2}$$

$$\sqrt{48} = \sqrt{(2\times2)\times(2\times2)\times3} = 2\times2\sqrt{3} = 4\sqrt{3}$$

$$\sqrt{63} = \sqrt{(3\times3)\times7} = 3\sqrt{7}$$

Rational and Irrational Numbers - In the test for rational and irrational numbers, if a surd has a

square root in the numerator, while the denominator is '1' or some other number, then the

number represented by the expression is 'irrational'.

examples of irrational surds:

$$\frac{3+\sqrt{3}}{2}, \quad \frac{5+\sqrt{7}}{3}, \quad \frac{6-\sqrt{2}}{5}$$