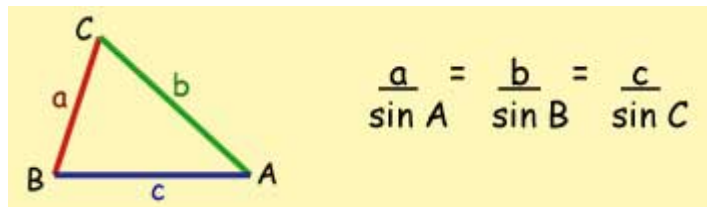


The Sine Rule

- Use either the right, or left hand equation.
- You are given 3 quantities and required to work out the 4 th.

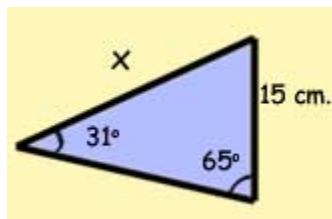
Manipulating the ratio - Take two ratios, cross multiply and rearrange to put the required quantity as the subject of the equation.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

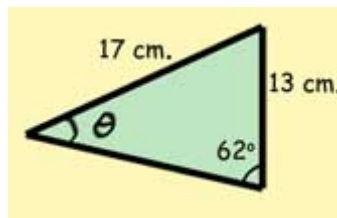
$$a \sin B = b \sin A$$

$$a = \frac{b \sin A}{\sin B} \quad b = \frac{a \sin B}{\sin A}$$

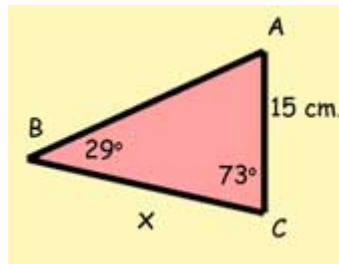
$$\sin B = \frac{b \sin A}{a} \quad \sin A = \frac{a \sin B}{b}$$

Example #1

$$\begin{aligned}\frac{15}{\sin 31^\circ} &= \frac{x}{\sin 65^\circ} \\ x \sin 31^\circ &= 15 \sin 65^\circ \\ x &= \frac{15 \sin 65^\circ}{\sin 31^\circ} \\ &= \frac{15 \times 0.9063}{0.5150} \\ &= 26.3971 \\ x &= \underline{26.40 \text{ cm}} \text{ (2 d.p.)}\end{aligned}$$

Example #2

$$\begin{aligned}\frac{17}{\sin 62^\circ} &= \frac{13}{\sin \theta} \\ 17 \sin \theta &= 13 \sin 62^\circ \\ \sin \theta &= \frac{13 \sin 62^\circ}{17} \\ &= \frac{13 \times 0.8829}{17} \\ &= 0.6752 \\ \theta &= 42.4697^\circ \\ \theta &= \underline{42.47^\circ} \text{ (2 d.p.)}\end{aligned}$$

Example #3

$$\begin{aligned}\text{angle } A &= 180^\circ - 29^\circ - 73^\circ \\ &= 78^\circ\end{aligned}$$

$$\frac{x}{\sin 78^\circ} = \frac{15}{\sin 29^\circ}$$

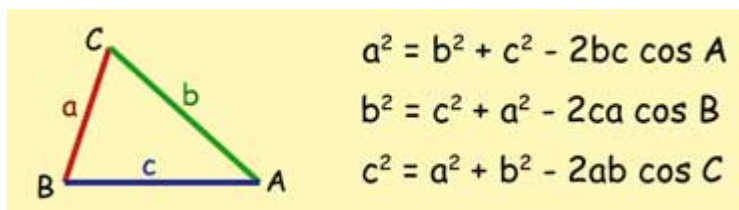
$$x \sin 29^\circ = 15 \sin 78^\circ$$

$$x = \frac{15 \sin 78^\circ}{\sin 29^\circ}$$

$$= \frac{15 \times 0.9781}{0.4848}$$

$$= 30.2630$$

$$\underline{x = 30.26 \text{ cm (2 d.p.)}}$$

The Cosine Rule

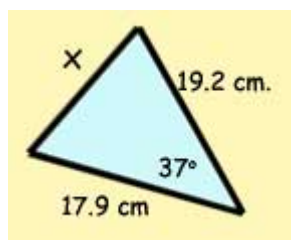
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

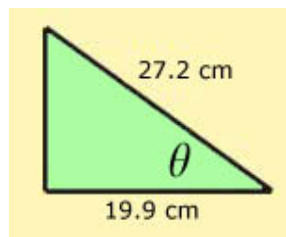
$$c^2 = a^2 + b^2 - 2ab \cos C$$

There are two problem types:

- You are given 2 sides + an included angle and required to work out the remaining side
- You are given all the sides and required to work out the angle.

Example #1

$$\begin{aligned}
 a^2 &= b^2 + c^2 - 2bc \cos A \\
 x^2 &= (19.2)^2 + (17.90)^2 - [2 \times 19.2 \times 17.9 \times \cos 37^\circ] \\
 &= (368.64) + (320.4) - (687.36) \times (0.7986) \\
 &= 140.1143 \\
 x &= 11.8369 \\
 &= \underline{11.84 \text{ cm}} \text{ (2 d.p.)}
 \end{aligned}$$

Example #2

$$\begin{aligned}
 a^2 &= b^2 + c^2 - 2bc \cos A \\
 \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\
 \cos \theta &= \frac{(23.6)^2 + (15.5)^2 - (21.8)^2}{2(23.6)(15.5)} \\
 &= \frac{(556.96) + (240.25) - (475.24)}{731.60} \\
 &= \frac{321.97}{731.60} = 0.4401 \\
 \theta &= 63.8897^\circ \\
 &= \underline{63.89^\circ} \text{ (2 d.p.)}
 \end{aligned}$$