Conventionally sequences have a **first term** or starting value, usually denoted by the letter '**a**'.

The **common difference** '**d**' is the difference between consecutive terms when the terms increase by a regular amount.

The difference change '**c**' is the change between consecutive differences

The **last term** in a sequence of '**n**' numbers is the **nth** term.

The **general term** is an expression in 'n' that can be used to calculate any term in the sequence.

'Common Difference' Sequences

The general term for term number '**n**' , common diff. '**d**' and first term '**a**' is:

dn + (a-d)

e.g. : 4.....9.....14.....19.....24.....29.....

a = 4, d = 5

the nth term is **dn + (a-d)** = 5n + (4-5) = 5n-1

n=7 , 7th term is (5x7)-1 = 34

example #1 - Find the nth term in this sequence : 13, 20, 27, 34, 41, 48 ...

a=13, d= 7

nth term = dn + (a-d) = 7n + (13-7) = 7n +6

example #2 - Find the nth term in this sequence : 11, 19, 27, 35, 43, 51 ...

a=11, d= 8

nth term = dn + (a-d) = 8n + (11-8) = 8n +3

example #3 - Find the nth term in this sequence : 9, 15, 21, 27, 33, 39 ...

a=9, d= 6

nth term = dn + (a-d) = 6n + (9-6) = 6n +3

'Changing Difference' Sequences

The general term for term number '**n**' , common diff. '**d**' , first term '**a**' and difference change '**c**'is:

$$a+d(n-1)+\frac{c}{2}(n-1)(n-2)$$

Example #1 - find the nth term of 3, 8, 14, 21, 29 .....

Writing the series with increases below:

3		8		14		21		29
	5		6		7		8	

remembering that the nth term is given by:

$$a+d(n-1)+\frac{c}{2}(n-1)(n-2)$$

1st term, 'a' =3

first difference 'd' = 5

difference increase 'c' = 1

$$\begin{aligned}\text{nth term} &= 3+5(n-1)+\frac{1}{2}(n-1)(n-2)\\ &= 3+5n-5+\frac{1}{2}(n-1)(n-2)\\ &= 3+5n-5+\frac{1}{2}(n^2-3n+2)\\ &= 5n-2+\frac{n^2}{2}-\frac{3n}{2}+\frac{2}{2}\\ &= \frac{7n}{2}-2+\frac{n^2}{2}+1\\ &= \frac{n^2}{2}+\frac{7n}{2}-1\end{aligned}$$

Example #2 - find the nth term of 5, 7, 10, 14, 19 .....

Writing the series with increases below:

5		7		10		14		19
	2		3		4		5	

remembering that the nth term is given by:

$$a + d(n-1) + \frac{c}{2}(n-1)(n-2)$$

1st term, 'a' =5

first difference 'd' = 2

difference increase 'c' = 1

$$\begin{aligned}
\text{nth term} &= 5 + 2(n-1) + \frac{1}{2}(n-1)(n-2) \\
&= 5 + 2n - 2 + \frac{1}{2}(n-1)(n-2) \\
&= 5 + 2n - 2 + \frac{1}{2}(n^2 - 3n + 2) \\
&= 5 + 2n - 2 + \frac{n^2}{2} - \frac{3n}{2} + \frac{2}{2} \\
&= 4 + \frac{n}{2} + \frac{n^2}{2} \\
&= \frac{n^2}{2} + \frac{n}{2} + 4
\end{aligned}$$