

Square Root The square root of a number is a number that must be squared (multiplied by itself) to give the original number.

$$\sqrt{9} = 3 \quad \sqrt{9} \times \sqrt{9} = 3 \times 3 = 9$$

$$\sqrt{25} = 5 \quad \sqrt{25} \times \sqrt{25} = 5 \times 5 = 25$$

$$\sqrt{64} = 8 \quad \sqrt{64} \times \sqrt{64} = 8 \times 8 = 64$$

$$\sqrt{144} = 12 \quad \sqrt{144} \times \sqrt{144} = 12 \times 12 = 144$$

Cube Root The cube root of a number is a number that must be cubed (multiplied by itself 3 times) to give the original number.

$$\sqrt[3]{8} = 2 \quad \sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 2 \times 2 \times 2 = 8$$

$$\sqrt[3]{27} = 3 \quad \sqrt[3]{27} \times \sqrt[3]{27} \times \sqrt[3]{27} = 3 \times 3 \times 3 = 27$$

$$\sqrt[3]{64} = 4 \quad \sqrt[3]{64} \times \sqrt[3]{64} \times \sqrt[3]{64} = 4 \times 4 \times 4 = 64$$

$$\sqrt[3]{125} = 5 \quad \sqrt[3]{125} \times \sqrt[3]{125} \times \sqrt[3]{125} = 5 \times 5 \times 5 = 125$$

The Index Laws An 'index' (plural 'indices') is a number written in small case to the upper right of a number to indicate the number's size. An index is sometimes called the 'power' of a number.

$$3^2 = 3 \times 3 = 9 \quad 4^3 = 4 \times 4 \times 4 = 64 \quad 5^4 = 5 \times 5 \times 5 \times 5 = 625$$

$$a^2 = a \times a \quad b^4 = b \times b \times b \times b \quad c^5 = c \times c \times c \times c \times c$$

The Index Law of Multiplication - Indices of **multiplied** terms are **added** to each other.

$$p^m \times p^n = p^{m+n}$$

$$3^3 \times 3^4 = 3^{3+4} = 3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187$$

$$4^2 \times 4^1 = 4^{2+1} = 4^3 = 4 \times 4 \times 4 = 64$$

$$5^2 \times 5^3 = 5^{2+3} = 5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$$

The Index Law of Division - Indices of **divided** terms are **subtracted** from each other.

$$\frac{p^m}{p^n} = p^{m-n}$$

$$\frac{3^3}{3^2} = 3^{3-2} = 3^1 = 3$$

$$\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 5 \times 5 = 25$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 4 \times 4 = 16$$

The Index Law of Raised Powers - Indices of terms in brackets, raised to another power have their indices **multiplied** by the index outside the brackets.

$$(p^m)^n = p^{m \times n} = p^{mn}$$

$$(3^2)^5 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3^{10}$$

$$(5^3)^2 = (5 \times 5 \times 5) \times (5 \times 5 \times 5) = 5^6$$

$$(4^4)^3 = (4 \times 4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4) = 4^{12}$$

The Index Law of Reciprocal Powers - The index of a reciprocal indexed quantity is multiplied by '-1' when turned upside down (inverted). Conversely, when a normal indexed quantity is inverted, its index is multiplied by '-1'.

$$\frac{1}{p^m} = p^{-m} \qquad p^n = \frac{1}{p^{-n}}$$

$$\frac{1}{5^3} = 5^{-3} \qquad \frac{1}{7^2} = 7^{-2} \qquad \frac{1}{13^9} = 13^{-9}$$

$$3^4 = \frac{1}{3^{-4}} \qquad 6^4 = \frac{1}{6^{-4}} \qquad 12^3 = \frac{1}{12^{-3}}$$

The Index Convention for Roots - The nth root of a number is the number to the power of 1/n.

$$\sqrt[n]{p} = p^{\frac{1}{n}} \qquad \sqrt[3]{5} = 5^{\frac{1}{3}} \qquad \sqrt[5]{11} = 11^{\frac{1}{5}}$$

$$\frac{1}{\sqrt[3]{5}} = 5^{-\frac{1}{3}} \qquad \frac{1}{\sqrt[6]{7}} = 7^{-\frac{1}{6}} \qquad \frac{1}{\sqrt[4]{10}} = 10^{-\frac{1}{4}}$$

Numbers to the power of zero - All number and quantities to the power of zero have a value of '1'.

$$p^0 = 1 \qquad 5^0 = 1 \qquad 7^0 = 1 \qquad 23^0 = 1 \qquad \text{etc...}$$

$$\frac{2^3}{2^3} = 2^3 \times 2^{-3} = 2^{3-3} = 2^0 \qquad \text{but } \frac{2^3}{2^3} = 1, \qquad \text{therefore } 2^0 = 1$$