

The Laws of Indices have been examined already with respect to 'number' under the heading 'powers & roots'.

However, in this section indices will be looked at in more depth, this time examples will use algebraic symbols.

The Laws of Indices

$$p^m \times p^n = p^{m+n}$$

$$\frac{p^m}{p^n} = p^{m-n}$$

$$(p^m)^n = p^{m \times n} = p^{mn}$$

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

$$\frac{1}{p^m} = p^{-m}$$

$$p^n = \frac{1}{p^{-n}}$$

$$p^0 = 1$$

Indices - Multiplication

remembering that:

$$p^m \times p^n = p^{m+n}$$

Examples

$$a^2 \times a^5 = a^7 \quad a^3 b^2 \times a^4 b^7 = a^7 b^9 \quad a^{-2} b^3 \times a^5 b^{-4} = a^3 b^{-1}$$

$$2a^3 b^2 c^{-3} \times 5a^3 b^{-2} c^2 = 10a^6 b^0 c^{-1} = 10a^6 c^{-1} \quad (b^0 = 1)$$

$$5a^2 b^{-7} c^{-2} \times 6a^{-2} b^5 c^3 = 30a^0 b^{-2} c^1 = 30b^{-2} c \quad (a^0 = 1, c^1 = c)$$

Indices - Division

remembering that:

$$\frac{p^m}{p^n} = p^{m-n}$$

Examples:

$$\frac{a^5}{a^6} = a^{-1}$$

$$\frac{a^3 b^5}{a^2 b^7} = a^1 b^{-2} = ab^{-2}$$

$$\frac{a^6 b^3}{a^7 b^{-5}} = a^{-1} b^{3-(-5)} = a^{-1} b^8$$

$$\frac{12a^3 b^2}{3a^{-4} b^4} = 4a^{3-(-4)} b^{-2} = 4a^7 b^{-2}$$

$$\frac{8a^4 b^2 c^{-7}}{2a^3 b^{-4} c^{-5}} = 4a^{4-3} b^{2-(-4)} c^{-7-(-5)} = 4ab^6 c^{-2}$$

Indices - Powers

remembering that:

$$(p^m)^n = p^{m \times n} = p^{mn}$$

Examples:

$$(a^3b^5)^3 = a^9b^{15} \quad (a^4b^2)^{-5} = a^{-20}b^{-10} \quad (a^{-2}b^4)^{-3} = a^6b^{-12}$$

$$(3a^2b)^3 = 27a^6b^3 \quad (2a^3b^4)^3 = 8a^9b^{12} \quad (4ab^3)^2 = 16a^2b^6$$

$$4(2ab^2)^3 = 4(8a^3b^6) = 32a^3b^6, \quad 3(4a^4b^3)^2 = 3(16a^8b^6) = 48a^8b^6$$

Indices - Roots and Reciprocals

remembering that:

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

and

$$\frac{1}{p^m} = p^{-m} \quad p^n = \frac{1}{p^{-n}}$$

Examples:

$$\frac{a^3}{b^4} = a^3b^{-4} \quad \frac{a^2}{b^{-3}c^2} = a^2b^3c^{-2} \quad \frac{ab^2c^3}{b^{-5}c^2} = ab^7c$$

$$\sqrt{\frac{a^2}{b^6}} = (a^2b^{-6})^{\frac{1}{2}} = a^{\frac{2}{2}}b^{-\frac{6}{2}} = ab^{-3} \quad \sqrt[3]{\frac{b^4}{a^{-7}}} = (a^7b^4)^{\frac{1}{3}} = a^{\frac{7}{3}}b^{\frac{4}{3}}$$

$$\frac{a^2b^{\frac{1}{3}}}{a^{\frac{1}{2}}b^3} = a^{2 - \frac{1}{2}} a^{-\frac{1}{2}} b^{-3} b^{\frac{1}{3}} = a^{\frac{4}{2} - \frac{1}{2}} a^{-\frac{1}{2}} b^{-\frac{9}{3} + \frac{1}{3}} = a^{\frac{3}{2}} b^{-\frac{8}{3}}$$