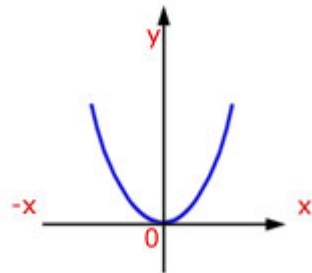
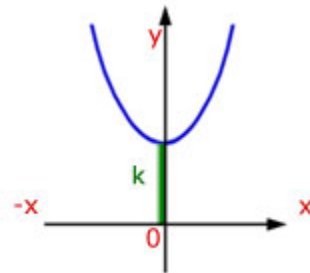


The function... $y = f(x) + k$

$$y = x^2$$



$$y = x^2 + k$$



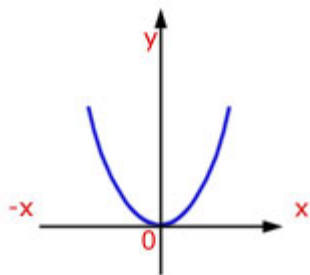
When $x = 0$, $y = k$. So the curve is moved (translated) by 'k' in the y-direction.

In vector terms the translation of the curve is

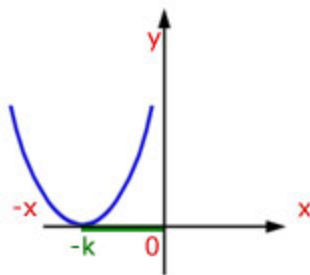
$$\begin{pmatrix} 0 \\ k \end{pmatrix}$$

The function... $y = f(x + k)$

$$y = x^2$$



$$y = (x + k)^2$$



This is best understood with an example.

Let k be equal to some number, say 3. Adding 3 into the original equation, we have:

$$y = f(x + 3)$$

$$y = (x + 3)^2$$

$$y = (x + 3)(x + 3)$$

$$\text{when } y = 0, x = -3$$

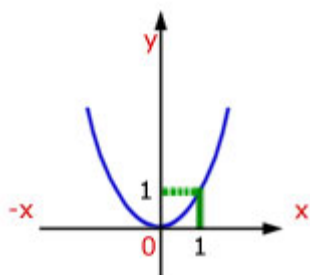
So the curve moves -3 to the left, to where $y=0$. That is $-k$ to the left.

In vector terms the translation of the curve is

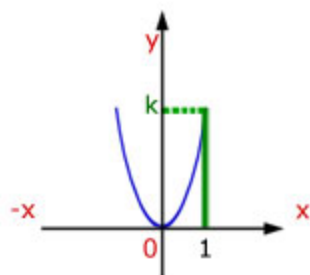
$$\begin{pmatrix} -k \\ 0 \end{pmatrix}$$

The function... $y = kf(x)$

$$y = x^2$$



$$y = k(x)^2$$



In our example, y increases by a factor of 'k' for every value of x .

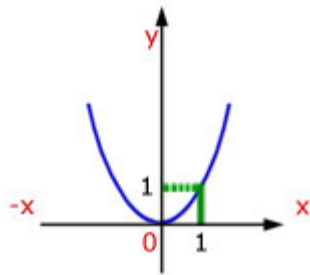
Example - let $k=5$

	$y = (x)^2$	$y = 5(x)^2$
$x = 1$	$y = (1)^2 = 1$	$y = 5(1)^2 = 5$
$x = 2$	$y = (2)^2 = 4$	$y = 5(2)^2 = 5(4) = 20$
$x = 3$	$y = (3)^2 = 9$	$y = 5(3)^2 = 5(9) = 45$

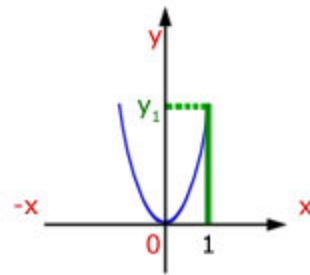
So for each value of x , the value of y is 5 times its previous value. The curve is stretched in the y -direction by a factor of 5. That is by a factor of k .

The function... $y = f(kx)$

$$y = x^2$$



$$y = (kx)^2$$



In the above, when $x=1$, $y=1$. However, in the second function when $x=1$, y is a higher value. Look at the example below for $x=1$ and other values of x .

Remember, in this function the constant 'k' multiplies the x-value **inside** the function.

Example #1 - let $k=4$

	$y = (x)^2$	$y = (4x)^2$
$x = 1$	$y = (1)^2 = 1$	$y = (4 \times 1)^2 = 16$
$x = 2$	$y = (2)^2 = 4$	$y = (4 \times 2)^2 = 64$
$x = 3$	$y = (3)^2 = 9$	$y = (4 \times 3)^2 = 144$

You will notice that the y-value jumps by a factor of 16 for each increasing x-value. The y-value increases by a factor of 4 squared.

With more complicated functions the value of y for a given value of x , increases once more, narrowing the curve in the x-direction (or stretching in the y-direction).

Example #2 a more complicated function with $k=4$

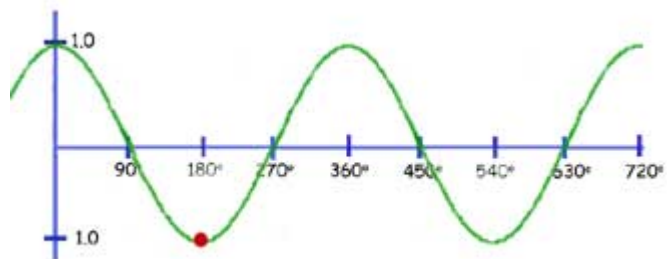
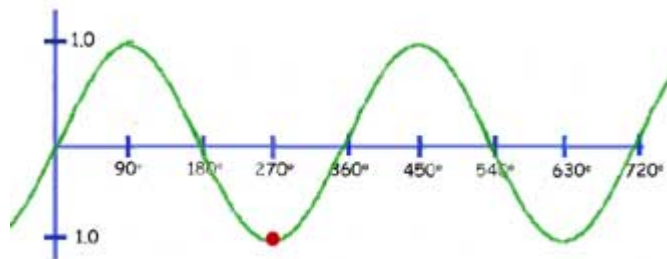
	$y = (x)^2 - 2(x) + 3$	$y = (4x)^2 - 2(4x) + 3$
	$y = x^2 - 2x + 3$	$y = 16x^2 - 8x + 3$
$x = 1$	$y = 1 - 2 + 3$	$y = 16(1) - 8(1) + 3$
$x = 1$	$y = 2$	$y = 16 - 8 + 3$
$x = 1$	$y = 2$	$y = 11$

The function... $y = \sin(x+k)$

Here the graph is translated by the value of k , to the **left**

So when $k=90$ deg. The curve moves horizontally 90 deg. (looking at the red dot, from 270 deg. to 180 deg.)

$$y = \sin(x)$$

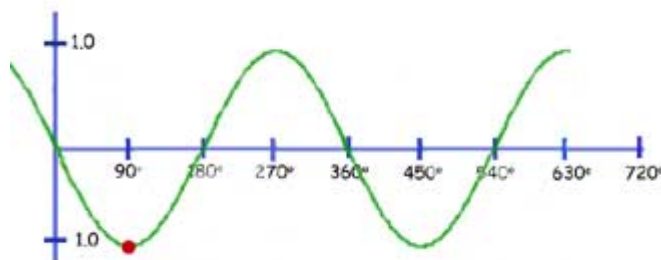
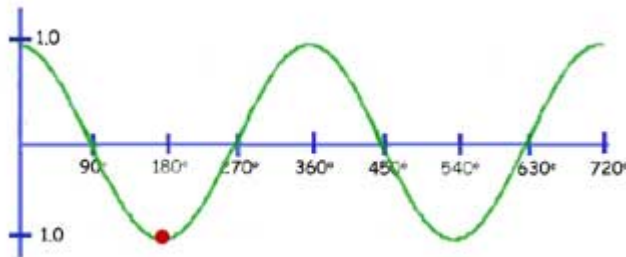


The function... $y = \cos(x+k)$

This is exactly the same as for the sine function.

The graph is translated by the value of k , to the **left**

So when $k=90$ deg. The curve moves horizontally 90 deg. (looking at the red dot, from 180 deg. to 90 deg.)



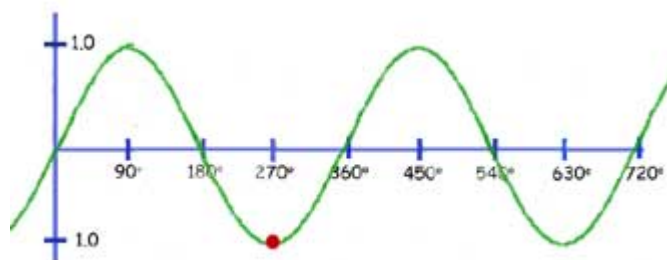
The function... $y = \sin(kx)$

Here the graph is squeezed horizontally (compressed) by a factor of k .

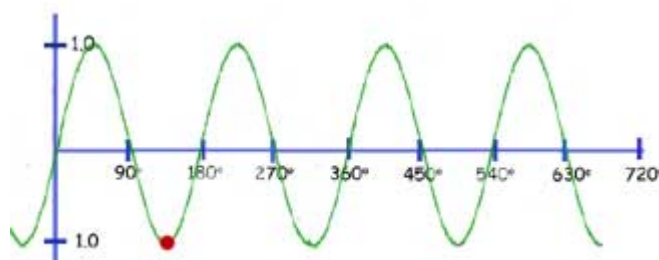
In our example below, $k = 2$. So one whole wavelength of 360° is reduced to 180° .

Conversely you may think of any value of x being halved (red spot reading changes from 270° to 135°)

$$y = \sin(x)$$



$$y = \sin(2x)$$



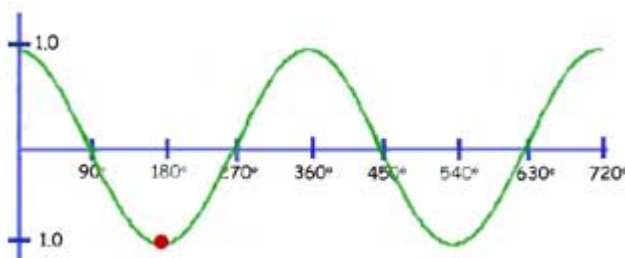
The function... $y = \cos(kx)$

As with the previous function, the graph is squeezed horizontally (compressed) by a factor of k .

In our example below, $k = 2$. So one whole wavelength of 360 deg. is reduced to 180 deg.

Conversely you may think of any value of x being halved (red spot reading changes from 180 deg. to 90 deg.).

$$y = \cos(x)$$



$$y = \cos(2x)$$

