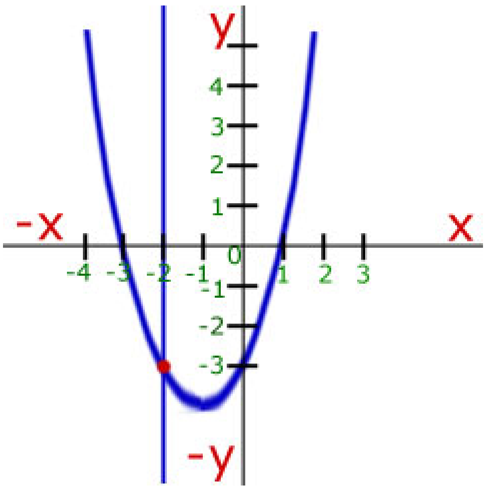
A 'straight line intersecting straight line' is dealt with in the 'simultaneous equations' section.

Vertical line intersecting a quadratic curve

Example Find the point of intersection when the vertical at x=-2 meets the curve,

$$y = x^2 + 2x - 3$$



Substitute the value of x=-2 into the quadratic equation to find y.

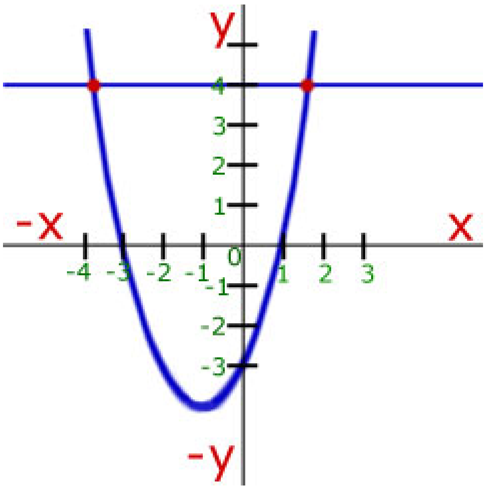
$$\begin{aligned} y &= x^2 + 2x - 3 \\ &= (-2)^2 + 2(-2) - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

hence the point of intersection is (-2, -3)

Horizontal line intersecting a quadratic curve

Example - Find the two points of intersection when the horizontal at y=4 meets the curve,

$$y = x^2 + 2x - 3$$



To find the two points, put one equation equal to the other, rearrange putting zero on one side and find the roots.

$$x^2 + 2x - 3 = 4$$
$$x^2 + 2x - 7 = 0$$

The roots are complex, therefore we use the quadratic equation formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, \quad b = 2, \quad c = -7$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 28}}{2}$$

$$= \frac{-2 \pm \sqrt{32}}{2}$$

$$= -1 \pm \frac{\sqrt{32}}{2} = -1 \pm \frac{\sqrt{2 \times 16}}{2} = -1 \pm 4\frac{\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

$$= -1 \pm 2\sqrt{2} = -1 \pm 2.828$$

$$\underline{x_1 = -1 + 2.828 = 1.828}$$

$$\underline{x_2 = -1 - 2.828 = -3.828}$$

check that x_1 and x_2 satisfy the quadratic and give $y = 4$

$$y_1 = x_1^2 + 2x_1 - 3 = (1.8)^2 + 2(1.8) - 3$$
$$= 3.2 + 3.6 - 3 = 3.8 \ \text{(approx.4)}$$
$$y_2 = x_2^2 + 2x_2 - 3 = (-3.8)^2 + 2(-3.8) - 3$$
$$= 14.4 - 7.6 - 3 = 3.8 \ \text{(approx.4)}$$

<u>The two points of intersection are (1.828, 4) and (-3.828, 4)</u>

N.B. the rounding of square roots makes the answers only approximate

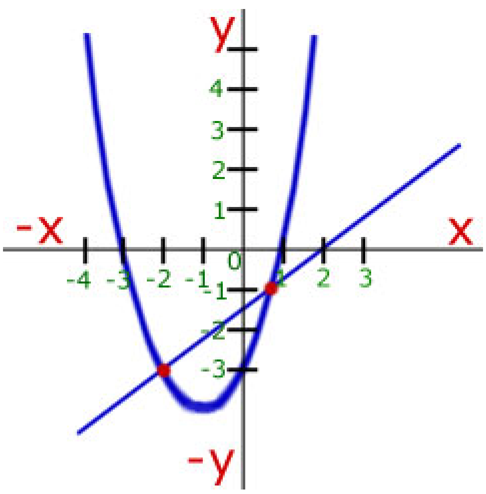
Angled straight line intersecting a quadratic curve

Example - Find the points of intersection when the straight line with equation,

$$y = \frac{3}{4}x - \frac{3}{2}$$

meets the curve,

$$y = x^2 + 2x - 3$$



As with the horizontal line intersection , the solution is to put one equation equal to the other, rearrange, put zero on one side and find the roots.

$$x^2+2x-3=\frac{3}{4}x-\frac{3}{2}$$

$$x^2+\frac{5}{4}x-\frac{3}{2}=0$$

$$a=1\,,\quad b=1.25\,,\quad c=-1.5$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1.25\pm\sqrt{(1.25)^2-4(1)(-1.5)}}{2(1)}$$

$$x=\frac{-1.25\pm\sqrt{1.56+6}}{2}=\frac{-1.25\pm\sqrt{7.76}}{2}$$

$$=\frac{-1.25\pm 2.79}{2}=-0.63\pm 1.39$$

$$\underline{x_1=-0.63+1.39=0.76}$$

using the straight line equation $\underline{y_1=0.75(0.76)-1.5=-0.93}$

$$\underline{x_2=-0.63-1.39=-1.99}$$

also $\underline{y_2=0.75(-1.99)-1.5=2.99}$

The two points of intersection are(0.76, -0.93) and (-1.99, 2.99)

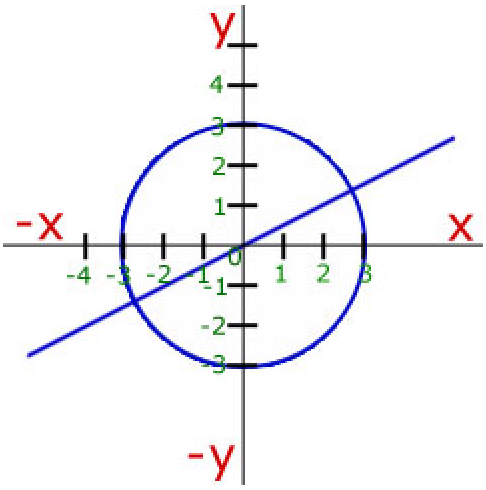
Straight line intersecting a circle

Example - Find the points of intersection when the straight line with equation,

$$y = \frac{x}{2}$$

meets the circle with equation,

$$x^2 + y^2 = 9$$



The solution is to take the y-value from the straight line equation and put it into the y-value of the circle equation. Then solve for x.

$$x^2 + \left(\frac{x}{2}\right)^2 = 9$$

$$x^2 + \frac{x^2}{4} = 9$$

$$4x^2 + x^2 = 36$$

$$5x^2 = 36\ , \quad x^2 = 7.2$$

$$\underline{x = \sqrt{7.2} = \pm\, 2.68}$$

$$\text{using} \quad y = \frac{x}{2}\ ,$$

$$\underline{y_1 = \frac{2.68}{2} = 1.34}$$

$$\underline{y_2 = \frac{-2.68}{2} = -1.34}$$

<u>The two points of intersection are(2.68, 1.34) and (-2.68, -1.34)</u>