Direct proportion - If y is proportional to x, this can be expressed as:

$$y \propto x$$

$$y = kx$$

where k is 'the constant of proportionality'

A very useful equation can be obtained if we consider two sets of values of x and y.

$$x_1 \quad y_1 \qquad y_1 = kx_1 \qquad \text{(i}$$
$$x_2 \quad y_2 \qquad y_2 = kx_2 \qquad \text{(ii}$$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2} \qquad \text{dividing (i by (ii}$$

There are 4 values here. Questions on direct proportion will give you 3 of these values and you will be required to work out the 4th.

Example #1 - A car travels 135 miles on 15 litres of petrol. How many miles will the car travel if it uses 25 litres?

$$y_1 = 135 \text{ miles }, \qquad x_1 = 15 \text{ litres}$$
$$y_2 = \; ? \qquad x_2 = 25 \text{ litres}$$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}$$

$$\frac{135}{y_2} = \frac{15}{25}$$

$$135 \times 25 = 15y_2 \qquad \text{cross multiplying}$$

$$\frac{135 \times 25}{15} = y_2$$

$$y_2 = \frac{9 \times 25}{1} = 225$$

Answer: 225 miles

Example #2 - The speed v of a rocket is directly proportional to the time t it travels. After 3 seconds its speed is 150 metres per second(m/s). How long after take-off will the speed reach 550m/s ?

$y_1 = 150$ m/s , $x_1 = 3$ seconds

$y_2 = 550$ m/s , $x_2 = ?$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}$$

$$\frac{150}{550} = \frac{3}{x_2}$$

$$150x_2 = 550 \times 3 \quad \text{cross multiplying}$$

$$x_2 = \frac{550 \times 3}{150} = 11$$

Answer: 11 seconds

Inverse proportion - If y is inversely proportional to x, this can be expressed as:

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

where k is 'the constant of proportionality'

Another very useful equation can be obtained if we consider two sets of values of x and y.

$$x_1 \quad y_1 \qquad y_1 = \frac{k}{x_1} \qquad (i$$

$$x_2 \quad y_2 \qquad y_2 = \frac{k}{x_2} \qquad (ii$$

$$\frac{y_1}{y_2} = \frac{x_2}{x_1} \qquad \text{dividing (i by (ii}$$

As with the equation for direct proportion, there are 4 values here. Questions on inverse proportion will give you 3 of these values and you will be required to work out the 4th.

Example - It is assumed that the value of a second-hand car is inversely proportional to its mileage. A car of value £1200 has a mileage of 50,000 miles. What will its value be when it has travelled 80,000 miles?

$$y_1 = £1200\,, \qquad x_1 = 50{,}000 \text{ miles}$$

$$y_2 = \; ? \qquad x_2 = 80{,}000 \text{ miles}$$

$$\frac{y_1}{y_2} = \frac{x_2}{x_1}$$

$$\frac{1200}{y_2} = \frac{80000}{50000}$$

$$1200 \times 50000 = 80000 y_2 \qquad \text{cross multiplying}$$

$$\frac{1200 \times 50000}{80000} = y_2$$

$$y_2 = \frac{1200 \times 5}{8} = 150 \times 5 = 750$$

Answer: £750

Variation - This covers a number of proportionalities involving 'square', 'square root','cube root', 'cube', inverse or a combination of these.

The first thing you need to do is to write down the proportion in symbols and then as an equation. Here are some examples:

'a' is proportional to 'b' squared	$a \propto b^2$	$a = kb^2$
'c' is inversely proportional to 'd' cubed	$c \propto \frac{1}{d^3}$	$c = \frac{k}{d^3}$
'e' varies as the square root of 'f'	$e \propto \sqrt{f}$	$e = k\sqrt{f}$
'g' is proportional to 'h' cubed	$g \propto h^3$	$g = kh^3$
'i' varies as the inverse of 'j' squared	$i \propto \frac{1}{j^2}$	$i = \frac{k}{j^2}$

In questions on variation you are usually given a pair of x,y values and a proportionality. You are given one further value of x or y, and are required to calculate the missing value.

- find the 'constant of proportionality'(k) using the first 'xy' values and write down the proportionality as an equation
- put the new value of x or y in the equation and solve for the missing value

Example - If the value of y is proportional to the square of x, and x is 4 when y is 96, what is the value of y when x is 13?

$$x_1 = 4 , \quad y_1 = 96$$
$$y \propto x^2 , \quad y_1 = kx_1^2$$
$$96 = k(4^2)$$
$$96 = k(16)$$
$$\frac{96}{16} = k , \quad k = \frac{96}{16} = 6$$
$$\underline{y = 6x^2}$$
$$x_2 = 13 \quad y_2 = kx_2^2$$
$$y_2 = 6(13^2) , \quad y_2 = 6(169)$$
$$\underline{y_2 = 1014}$$

Curve Sketching - Try to remember the proportionality that matches the shape of the curve.

Algebraic Proportion

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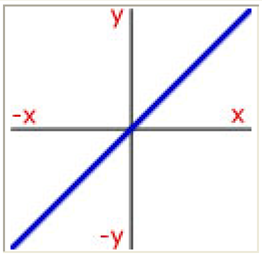
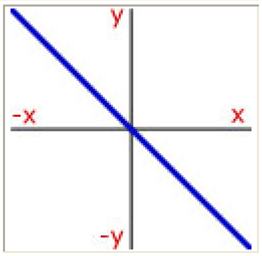
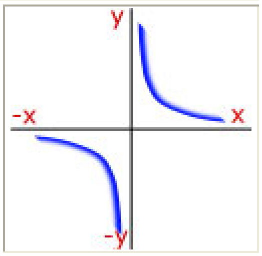
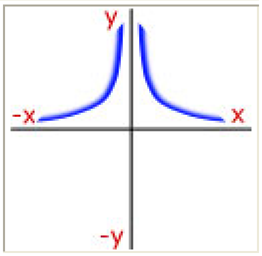
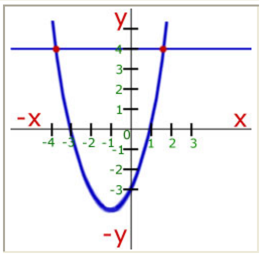
$y \propto x$	
$y \propto -x$	
$y \propto \frac{1}{x}$	
$y \propto \frac{1}{x^2}$	
$y \propto x^2$	
$y \propto \sqrt{x}$