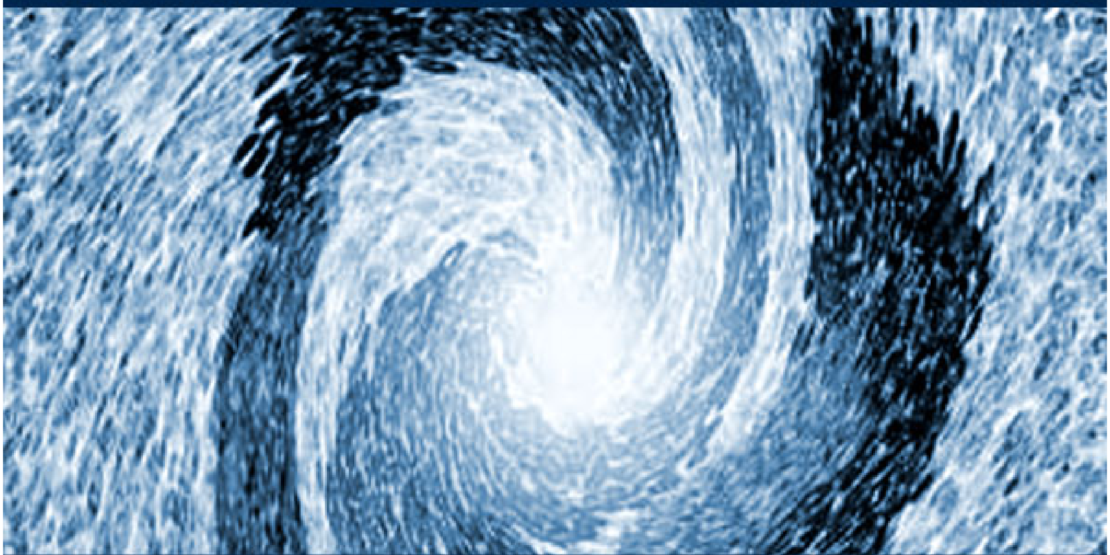
GCSE MATHS TUTOR

Revision Guide



PART ONE

NUMBER

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Contents

Number Types

These are the main number types:

integers, natural, prime, rational, irrational, square, surds, real, square, factors

INTEGERS

-3, -2, -1, 0, 1, 2, 3 ... Integers - zero, positive and negative WHOLE numbers.

NATURAL NUMBERS

Natural numbers are POSITIVE integers   2, 5, 8, 93, 37, 29 ...

PRIME NUMBERS

Prime Numbers can only be divided by themselves and '1' .

3, 5, 7, 11, 13, 17, 23, 29 ....

RATIONAL NUMBERS

All Rational numbers can be WRITTEN AS FRACTIONS, where the top number (numerator) and bottom number(denominator) are whole numbers

5/6, 9/5, 11/3, 12/7, 8/6, 24/10...

Where a decimal number is concerned, there is some pattern in the numbers after the decimal point. in this case we call the decimal 'recurring'. By convention the repeated numbers have a dot placed above them.

IRRATIONAL NUMBERS

Irrational Numbers cannot be written as fractions. Irrational numbers have no pattern in the numbers after the decimal point. The numbers go on randomly.

e.g. 5.93847562100291837441 21

SQUARE NUMBERS

A square number is produced when a number is multiplied by itself.

25, 16, 36, 9 are squares of 5, 4, 6, 3 respectively

SURDS

Surds are positive or negative numbers with a square root sign infront of them.

REAL NUMBERS

These are all the types of number discussed here. If you take your studies further to A-level, you will learn about VIRTUAL NUMBERS. These are to do with the square root of -1.

FACTORS

Factors are numbers that can divide exactly into other numbers without leaving a remainder. For example, 5 is a factor of 35. It divides into it exactly 7 times.

Lowest Common Multiple (L.C.M.)

The LCM(lowest common multiple) of two or more numbers is the smallest number that each will divide into exactly.

the LCM of 2, 3, 5 is...........30 (2 x 3 x 5)

example#1 - find the LCM of the following: 30, 64

First find the factors by dividing the numbers by prime numbers, 2, 3, 5 etc. to reduce them to '1',

2	30
3	15
5	5
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

30 = 3 x 5 x 2

64 = 2 x 2 x 2 x 2 x 2 x 2

The number that both will go into is all these factors multiplied together.

(3 x 5 x 2) x (2 x 2 x 2 x 2 x 2 x 2)

However, this is **not** the lowest common multiple.

To make the multiple smaller we can lose one of the 2's and still have each of the numbers divide into it(because the factor '2' is common to both numbers). .

3 x 5 x 2 x 2 x 2 x 2 x 2 x 2

So the LCM is of 30 & 64 is ....(3 x 5 x 2 x 2 x 2 x 2 x 2 x 2) = 960 example#2 - find the LCM of the following: 54, 96

First find the factors by dividing the numbers by prime numbers, 2, 3, 5 etc. to reduce the numbers to '1',

2	54
3	27
3	9
3	3
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

54 = 3 x 3 x 3 x 2

96 = 3 x 2 x 2 x 2 x 2 x 2

The number that both will go into is all these factors multiplied together.

(3 x 3 x 3 x 2) x (3 x 2 x 2 x 2 x 2 x 2)

However, this is **not** the lowest common multiple.

To make the multiple smaller we can lose one of the 3's and one of the 2's and still have each of the numbers divide into it(because these factors '3 x 2' are common to both numbers).

3 x 3 x 3 x 2 x 2 x 2 x 2 x 2

So the LCM of 54 & 96 is ... (3 x 3 x 3 x 2 x 2 x 2 x 2 x 2) = 864 example#3 - find the LCM of the following: 36, 98

First find the factors by dividing the numbers in turn by prime numbers, 2, 3, 5 etc. to reduce them to '1',

2	36
2	18
3	9
3	3
	1

2	98
7	49
7	7
	1

36 = 2 x 2 x 3 x 3

98 = 2 x 7 x 7

The number that both will go into is all these factors multiplied together.

(3 x 3 x 2 x 2) x (2 x 7 x 7)

However, this is **not** the lowest common multiple.

To make the multiple smaller we can lose one of the 2's and still have each of the numbers divide into it(because this factor '2' is common to both numbers).

(3 x 3 x 2 x 2 x 7 x 7)

So the LCM of 36 & 98 is .....(3 x 3 x 2 x 2 x 7 x 7) = 1764

Highest Common Factor (H.C.F.)

The HCF(highest common factor) of two (or more) numbers is the highest number that will divide into each of them exactly.

the HCF of 15, 25, 40 is 5

example#1 - find the HCF of the following: 36, 50

First find the factors by dividing the numbers by prime numbers, 2, 3, 5 etc. to reduce them to '1',

2	36
2	18
3	9
3	3
	1

2	50
5	25
5	5
	1

36 = 3 x 3 x 2 x 2

50 = 2 x 5 x 5

The number that divides into both is 2 .

36 = 3 x 3 x 2 x 2 50 = 2 x 5 x 5

(divides 3 x 3 x 2 =18 times) (divides 5 x 5 = 25 times)

So the HCF of 36 & 50 is ...2

example#2 - find the HCF of the following: 54, 96

First find the factors by dividing the numbers by prime numbers, 2, 3, 5 etc. to reduce them to '1',

2	54
3	27
3	9
3	3
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

54 = 2 x 3 x 3 x 3

96 = 2 x 2 x 2 x 2 x 2 x 3

The number that divides into both is 2 x 3 .

54 = 2 x 3 x 3 x 3

(divides 3 x 3 = 9 times)

96 = 2 x 2 x 2 x 2 x 2 x 3

(divides 2 x 2 x 2 x 2 = 16 times)

So the HCF of 54 & 96 is ...2 x 3 = 6

example#3 - find the HCF of the following: 48, 256

First find the factors by dividing the numbers by prime numbers, 2, 3, 5 etc. to reduce them to '1',

2	48
2	24
2	12
2	6
3	3
	1

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

48 = 2 x 2 x 2 x 2 x 3

256 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

The number that divides into both is 2 x 2 x 2 x 2 .

48 =2 x 2 x 2 x 2 x 3

(divides 3 times)

256 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

(divides 2 x 2 x 2 x 2 = 16 times)

So the HCF of 48 & 256 is .... 2 x 2 x 2 x 2 = 16

Operators

+		sum(or total) of		addition
-		difference between		subtraction
x		product of		multiplication
/		quotient of		division

Rules for Multiplication & Division

(+)x(+) = +

....

(+)x(−) = −

....

(−)x(+) = −

....

(−)x(−) = +

a '+' multiplied by a '+' gives a '+'

a '+ ' multiplied by a ' - ' gives a ' - '

a ' - ' multiplied by a '+' also gives a ' - '

a ' - ' multiplied by a ' - ' gives a '+'

$$\frac{+}{+} = +$$

....

$$\frac{+}{-} = -$$

....

$$\frac{-}{+} = -$$

....

$$\frac{-}{-} = +$$

a '+' divided by a '+' gives a '+'

a '+ ' divided by a ' - ' gives a ' - '

a ' - 'divided by a '+' also gives a ' - '

a ' - ' divided by a ' - ' gives a '+'

Examples

(+3)(-4)= -12	(-2)/(+9)= -(2/9)
(-5)(-2)= +10	(+3)/(-5)= -(3/5)
(+7)(+2)= +14	(+6)/(+7)= +(6/7)
(-9)(+5)= -45	(-5)/(-2)= +(5/2)

Operator Precedence ....This is the order in which calculations are performed.

BIDMAS - **b**rackets, **i**ndices, **d**ivision, **m**ultiplication, **a**dd, **s**ubtract

example #1

4 + 7 x 2 - 3(5 + 9)

4 + 7 x 2 - 3(14).......work out what is inside the brackets

4 + 14 - 42...............do the multiplication 7 x 2 and 3 x 14

28 - 42 ....add the 4 and 14

-14 ....subtract the 42

example #2

7(12-9) x (3+4) - 7 + 8 x 20/5

7(12-9) x (3+4) - 7 + 8 x 20/5.....work out what is inside the brackets

7(3) x (7) - 7 + 8 x 20/5................division next

7 x 3 x (7) - 7 + 8 x 4....................multiplication

21 x 7 - 7 + 32.............................more multiplication

147 - 7 +32..................................addition

179 -7...subtraction

172

example #3

3 + 9(6 - 2) - 12/3 + 4 x 7.....work out what is inside the brackets

3 + 9(4) - 12/3 + 4 x 7................division next

3 + 9 x 4 - 4 + 4 x 7...................multiplication

3 + 36 - 4 + 4 x 7.............................more multiplication

3 + 36 - 4 + 28 ....addition

67 - 4..subtraction

63

Powers & Roots

Square Root The square root of a number is a number that must be squared (multiplied by itself) to give the original number.

$\sqrt{9} = 3 \qquad \sqrt{9} \times \sqrt{9} = 3 \times 3 = 9$

$\sqrt{25} = 5 \qquad \sqrt{25} \times \sqrt{25} = 5 \times 5 = 25$

$\sqrt{64} = 8 \qquad \sqrt{64} \times \sqrt{64} = 8 \times 8 = 64$

$\sqrt{144} = 12 \qquad \sqrt{144} \times \sqrt{144} = 12 \times 12 = 144$

Cube Root The cube root of a number is a number that must be cubed (multiplied by itself 3 times) to give the original number.

$\sqrt[3]{8} = 2 \qquad \sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 2 \times 2 \times 2 = 8$

$\sqrt[3]{27} = 3 \qquad \sqrt[3]{27} \times \sqrt[3]{27} \times \sqrt[3]{27} = 3 \times 3 \times 3 = 27$

$\sqrt[3]{64} = 4 \qquad \sqrt[3]{64} \times \sqrt[3]{64} \times \sqrt[3]{64} = 4 \times 4 \times 4 = 64$

$\sqrt[3]{125} = 5 \qquad \sqrt[3]{125} \times \sqrt[3]{125} \times \sqrt[3]{125} = 5 \times 5 \times 5 = 125$

The Index Laws An 'index' (plural 'indices') is a number written in small case to the upper right of a number to indicate the number's size. An index is sometimes called the 'power' of a number.

$3^2 = 3 \times 3 = 9 \qquad 4^3 = 4 \times 4 \times 4 = 64 \qquad 5^4 = 5 \times 5 \times 5 \times 5 = 625$

$a^2 = a \times a \qquad b^4 = b \times b \times b \times b \qquad c^5 = c \times c \times c \times c \times c$

<u>The Index Law of Multiplication</u> - Indices of **multiplied** terms are **added** to eachother.

$$p^m \times p^n = p^{m+n}$$

$$3^3 \times 3^4 = 3^{3+4} = 3^7 = 3\times3\times3\times3\times3\times3\times3 = 2187$$

$$4^2 \times 4^1 = 4^{2+1} = 4^3 = 4\times4\times4 = 64$$

$$5^2 \times 5^3 = 5^{2+3} = 5^5 = 5\times5\times5\times5\times5 = 3125$$

<u>The Index Law of Division</u> - Indices of **divided** terms are **subtracted** from eachother.

$$\frac{p^m}{p^n} = p^{m-n}$$

$$\frac{3^3}{3^2} = 3^{3-2} = 3^1 = 3$$

$$\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 5\times5 = 25$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 4\times4 = 16$$

<u>The Index Law of Raised Powers</u> - Indices of terms in brackets, raised to another power have their indices **multiplied** by the index outside the brackets.

$$(p^m)^n = p^{m\times n} = p^{mn}$$

$$(3^2)^5 = (3\times3)\times(3\times3)\times(3\times3)\times(3\times3)\times(3\times3) = 3^{10}$$

$$(5^3)^2 = (5\times5\times5)\times(5\times5\times5) = 5^6$$

$$(4^4)^3 = (4\times4\times4\times4)\times(4\times4\times4\times4)\times(4\times4\times4\times4) = 4^{12}$$

The Index Law of Reciprocal Powers - The index of a reciprocal indexed quantity is multiplied by '**-1**' when turned upside down(inverted). Conversely, when a normal indexed quantity is inverted, its index is multiplied by '-1'.

$$\frac{1}{p^m} = p^{-m} \qquad p^n = \frac{1}{p^{-n}}$$

$$\frac{1}{5^3} = 5^{-3} \qquad \frac{1}{7^2} = 7^{-2} \qquad \frac{1}{13^9} = 13^{-9}$$

$$3^4 = \frac{1}{3^{-4}} \qquad 6^4 = \frac{1}{6^{-4}} \qquad 12^3 = \frac{1}{12^{-3}}$$

The Index Convention for Roots - The nth root of a number is the number to the power of 1/n.

$$\sqrt[n]{p} = p^{\frac{1}{n}} \qquad \sqrt[3]{5} = 5^{\frac{1}{3}} \qquad \sqrt[5]{11} = 11^{\frac{1}{5}}$$

$$\frac{1}{\sqrt[3]{5}} = 5^{-\frac{1}{3}} \qquad \frac{1}{\sqrt[6]{7}} = 7^{-\frac{1}{6}} \qquad \frac{1}{\sqrt[4]{10}} = 10^{-\frac{1}{4}}$$

Numbers to the power of zero - All number and quantities to the power of zero have a value of '1'.

$$p^0 = 1 \qquad 5^0 = 1 \qquad 7^0 = 1 \qquad 23^0 = 1 \qquad \text{etc...}$$

$$\frac{2^3}{2^3} = 2^3 \times 2^{-3} = 2^{3-3} = 2^0 \qquad \text{but } \frac{2^3}{2^3} = 1, \qquad \text{therefore } 2^0 = 1$$

Growth & Decay

All problems on growth & decay - full name 'exponential growth & decay' use the formula below, where the time variable 'n' is in the index.

$$A = A_o\left(1 \pm \frac{r}{100}\right)^n$$

A_o - initial amount

A - new amount

r - % change

n - time(seconds, hours, days etc.)

$\pm$ - '+' for growth, '-' for decay

Example #1 - £5000 is invested in a savings bond, which pays 7% p.a.

How much will the bond be worth after 5 years?

$$A = A_o\left(1 + \frac{r}{100}\right)^n$$

A_o - £5000

A - new amount

r - 7%

n - 5 years

$\pm$ - '+' for growth

$$A = 5000\left(1 + \frac{7}{100}\right)^5$$

$$A = 5000(1.07)^5$$

$$A = £7,012.76$$

Example #2 - The level of activity of a radio-active source decreases by 5% per hour. If the activity is 1500 counts per second, what will it be 12 hours later?

$$A = A_o\left(1 - \frac{r}{100}\right)^n$$

A_o - 1500 counts/sec.

A - new amount

r - 5%

n - 12 hours

$\pm$ - '-' for decay

$$A = 1500\left(1 - \frac{5}{100}\right)^{12}$$

$$A = 1500\left(0.95\right)^{12}$$

$$A = 1500(0.540)$$

$$A = 810.5 \text{ counts/sec.}$$

Example #3 - A bee-hive increases its population of bees by 8% per year.

How many bees will there be in the hive in 5 years time?

$$A = A_o\left(1 + \frac{r}{100}\right)^n$$

A_o - 6,500 bees

A - new amount

r - 8%

n - 5 years

$\pm$ - '+' for decay

$$A = 6500\left(1 + \frac{8}{100}\right)^5$$

$$A = 6500(1.08)^5$$

$$A = 9550.63$$

$$A = 9550 \text{ bees (rounding down)}$$

Surds

Surds

Surds are mathematical expressions containing square roots. However, it must be emphasized that

the square roots are 'irrational' i.e. they do not result in a whole number, a terminating decimal or a

recurring decimal.

The rules governing surds are taken from the Laws of Indices.

rule #1

$$\sqrt{c} \times \sqrt{d} = \sqrt{c \times d} \qquad c^{1/2} \times d^{1/2} = (c \times d)^{1/2}$$

examples

$$\sqrt{5} \times \sqrt{3} = \sqrt{3 \times 5} = \sqrt{15}$$

$$\sqrt{12} \times \sqrt{5} = \sqrt{12 \times 5} = \sqrt{60}$$

$$\sqrt{7} \times \sqrt{2} = \sqrt{7 \times 2} = \sqrt{14}$$

rule #2

$$\frac{\sqrt{c}}{\sqrt{d}} = \sqrt{\frac{c}{d}} \qquad \frac{c^{1/2}}{d^{1/2}} = \left(\frac{c}{d}\right)^{1/2}$$

examples

$$\frac{\sqrt{15}}{\sqrt{5}} = \sqrt{\frac{15}{5}} = \sqrt{3}$$

$$\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$$

$$\frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}$$

Some Useful Expressions

expression #1

$$
\begin{aligned}
(c+d)^2 &= (c+d)(c+d) \\
&= c^2 + cd + cd + d^2 \\
&= c^2 + 2cd + d^2
\end{aligned}
$$

$$
\begin{aligned}
(c+\sqrt{d})^2 &= (c+\sqrt{d})(c+\sqrt{d}) \\
&= c^2 + c\sqrt{d} + c\sqrt{d} + d \\
&= c^2 + 2c\sqrt{d} + d
\end{aligned}
$$

$$
\begin{aligned}
(5+\sqrt{2})^2 &= (5+\sqrt{2})(5+\sqrt{2}) \\
&= 5^2 + 5\sqrt{2} + 5\sqrt{2} + 2 \\
&= 5^2 + 10\sqrt{2} + 2 \\
&= 25 + 10\sqrt{2} + 2 \\
&= 27 + 10\sqrt{2}
\end{aligned}
$$

expression #2 - (the difference of two squares)

$$(c+d)(c-d) = c^2 + cd - cd + d^2$$
$$= c^2 - d^2$$

$$(c+\sqrt{d})(c-\sqrt{d}) = c^2 + c\sqrt{d} - c\sqrt{d} - (\sqrt{d})^2$$
$$= c^2 - d$$

$$
\begin{aligned}
(7+\sqrt{3})(7-\sqrt{3}) &= 7^2 - (\sqrt{3})^2 \\
&= 49 - 3 \\
&= 46
\end{aligned}
$$

<u>Rationalising Surds</u> - This is a way of modifying surd expressions so that the square root is in the numerator of a fraction and not in the denominator.

The method is to multiply the top and bottom of the fraction by the square root.

$\frac{5}{\sqrt{3}}$, $\frac{5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{5\times\sqrt{3}}{3}$

$\frac{8}{\sqrt{5}}$, $\frac{8\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{8\times\sqrt{5}}{5}$

$\frac{7}{\sqrt{2}}$, $\frac{7\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{7\times\sqrt{2}}{2}$

<u>Rationalising expressions using the 'difference of two squares'</u>

Remembering that : .... $(c+\sqrt{d})(c-\sqrt{d}) = c^2 - d$ .....from 'useful expressions' above.

Example #1 – simplify

$$\frac{2+\sqrt{5}}{2-\sqrt{5}}$$

multiplying top and bottom by $(2+\sqrt{5})$

$$\frac{(2+\sqrt{5})(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}$$

$$=\frac{2^2+2\sqrt{5}+2\sqrt{5}+5}{2^2-(\sqrt{5})^2}$$

$$=\frac{4+4\sqrt{5}+5}{4-5}$$

$$=\frac{9+4\sqrt{5}}{-1}$$

$$=-9-4\sqrt{5}$$

Example #2 – rationalise

$$\frac{2}{6+3\sqrt{5}}$$

multiply top and bottom by $6-3\sqrt{5}$

$$\frac{2(6-3\sqrt{5})}{(6+3\sqrt{5})(6-3\sqrt{5})}$$

$$=\frac{12-6\sqrt{5})}{6^2-(3\sqrt{5})^2}$$

$$=\frac{12-6\sqrt{5}}{6^2-(3\sqrt{5})^2}=\frac{12-6\sqrt{5}}{6^2-(3\times3\times\sqrt{5}\times\sqrt{5})}$$

$$=\frac{12-6\sqrt{5}}{6^2-(9\times5)}=\frac{12-6\sqrt{5}}{36-45}=\frac{12-6\sqrt{5}}{-9}=-\frac{(12-6\sqrt{5})}{9}$$

$$=-\frac{3(4-2\sqrt{5})}{9}=-\frac{(4-2\sqrt{5})}{3}=\frac{-4+2\sqrt{5}}{3}$$

<u>Reduction of Surds</u> - This is a way of making the square root smaller by examining its squared

factors and removing them.

$$\sqrt{18} = \sqrt{(3\times3)\times2} = \sqrt{3\times3}\times\sqrt{2} = 3\sqrt{2}$$

$$\sqrt{48} = \sqrt{(2\times2)\times(2\times2)\times3} = 2\times2\sqrt{3} = 4\sqrt{3}$$

$$\sqrt{63} = \sqrt{(3\times3)\times7} = 3\sqrt{7}$$

<u>Rational and Irrational Numbers</u> - In the test for rational and irrational numbers, if a surd has a

square root in the numerator, while the denominator is '1' or some other number, then the

number represented by the expression is 'irrational'.

examples of irrational surds:

$$\frac{3+\sqrt{3}}{2}\ ,\quad \frac{5+\sqrt{7}}{3}\ ,\quad \frac{6-\sqrt{2}}{5}$$

Sequences

Conventionally sequences have a **first term** or starting value, usually denoted by the letter '**a**'.

The **common difference** '**d**' is the difference between consecutive terms when the terms increase by a regular amount.

The difference change '**c**' is the change between consecutive differences

The **last term** in a sequence of '**n**' numbers is the **nth** term.

The **general term** is an expression in 'n' that can be used to calculate any term in the sequence.

'Common Difference' Sequences

The general term for term number '**n**' , common diff. '**d**' and first term '**a**' is:

$$dn + (a-d)$$

e.g. : 4.....9.....14.....19.....24.....29.....

$a = 4$, $d = 5$

the nth term is **dn + (a-d)** = $5n + (4-5) = 5n-1$

$n=7$, 7th term is $(5x7)-1 = 34$

<u>example #1</u> - Find the nth term in this sequence : 13, 20, 27, 34, 41, 48 ...

a=13, d= 7

nth term = dn + (a-d) = 7n + (13-7) = <u>7n +6</u>

<u>example #2</u> - Find the nth term in this sequence : 11, 19, 27, 35, 43, 51 ...

a=11, d= 8

nth term = dn + (a-d) = 8n + (11-8) = <u>8n +3</u>

<u>example #3</u> - Find the nth term in this sequence : 9, 15, 21, 27, 33, 39 ...

a=9, d= 6

nth term = dn + (a-d) = 6n + (19-6) = <u>6n +13</u>

'Changing Difference' Sequences

The general term for term number '**n**' , common diff. '**d**' , first term '**a**' and differnce change '**c**'is:

$$a + d(n-1) + \frac{c}{2}(n-1)(n-2)$$

Example #1 - find the nth term of 3, 8, 14, 21, 29 ....

Writing the series with increases below:

3		8		14		21		29
	5		6		7		8	

remembering that the nth term is given by:

$$a + d(n-1) + \frac{c}{2}(n-1)(n-2)$$

1st term, 'a' =3

first difference 'd' = 5

difference increase 'c' = 1

$$\text{nth term } = 3 + 5(n-1) + \frac{1}{2}(n-1)(n-2)$$

$$= 3 + 5n - 5 + \frac{1}{2}(n-1)(n-2)$$

$$= 3 + 5n - 5 + \frac{1}{2}(n^2 - 3n + 2)$$

$$= 5n - 2 + \frac{n^2}{2} - \frac{3n}{2} + \frac{2}{2}$$

$$= \frac{7n}{2} - 2 + \frac{n^2}{2} + 1$$

$$= \frac{n^2}{2} + \frac{7n}{2} - 1$$

Example #2 - find the nth term of 3, 8, 14, 21, 29 ....

Writing the series with increases below:

5		7		10		14		19
	2		3		4		5	

remembering that the nth term is given by:

$$a + d(n-1) + \frac{c}{2}(n-1)(n-2)$$

1st term, 'a' =5

first difference 'd' = 2

difference increase 'c' = 1

$$\text{nth term } = 5 + 2(n-1) + \frac{1}{2}(n-1)(n-2)$$

$$= 5 + 2n - 2 + \frac{1}{2}(n-1)(n-2)$$

$$= 5 + 2n - 2 + \frac{1}{2}(n^2 - 3n + 2)$$

$$= 5 + 2n - 2 + \frac{n^2}{2} - \frac{3n}{2} + \frac{2}{2}$$

$$= 4 - n + \frac{n^2}{2}$$

$$= \frac{n^2}{2} - n + 4$$

Fractions

A fraction is made from two components: the numerator (on the top) and the denominator (on the bottom)

$$\frac{\text{numerator}}{\text{denominator}}$$

Fraction Types:

$$\frac{3}{4}, \frac{1}{2}, \frac{2}{3}, \frac{4}{5}$$

$$\frac{5}{3}, \frac{7}{2}, \frac{6}{5}, \frac{12}{4}$$

$$\frac{2}{3}, \frac{4}{6}, \frac{12}{18}, \frac{40}{60}$$

ordinary: numerator less than denominator

vulgar: numerator greater than denominator

equivalent: all the fractions can be cancelled to one simple fraction

Fraction Addition - To add two fractions you must first find their common denominator. Then convert each to the new denominator and add the numerators.

Look at this example.

$$\frac{4}{5}+\frac{2}{3}$$

The common denominator is found by multiplying the two denominators together. In this case, multiply the 5 by the 3. This gives **15**.

Now convert each factor to 15 ths by dividing the denominator of each into 15 and multiplying the result by each numerator.

$$\frac{4}{5}+\frac{2}{3}$$

$$\frac{12+10}{15}=\frac{22}{15}=1\frac{7}{15}$$

- 5 goes into 15 three times. 3 goes into 15 five times.
- Multiply the numerator of the first fraction 4, by three to get 12.
- Multiply the numerator of the second fraction 2, by five to get 10.
- Write the two results over the denominator 15, and add.

more examples...

$$\frac{3}{4}+\frac{1}{2}$$

$$\frac{6+4}{8}=\frac{10}{8}=1\frac{2}{8}=1\frac{1}{4}$$

$$\frac{5}{6}+\frac{2}{5}$$

$$\frac{25+12}{30}=\frac{37}{15}=2\frac{7}{15}$$

$$\frac{1}{7}+\frac{4}{9}$$

$$\frac{9+28}{63}=\frac{37}{63}$$

Fraction Subtraction - This is exactly the same as for addition except that the numbers above the common denominator are subtracted from eachother.

$$\frac{4}{5}-\frac{2}{3}$$
$$\frac{12-10}{15}=\frac{2}{15}$$

$$\frac{1}{2}-\frac{3}{7}$$
$$\frac{7-6}{14}=\frac{1}{14}$$

$$\frac{9}{10}-\frac{5}{9}$$
$$\frac{81-50}{90}=\frac{31}{90}$$

Fraction Multiplication - Here the numerators are multiplied along the top, while the denominators are multiplied along the bottom.

$$\frac{5}{9}\times\frac{7}{12}=\frac{5\times7}{9\times12}=\frac{35}{108}$$

$$\frac{3}{4}\times\frac{1}{10}=\frac{3\times1}{4\times10}=\frac{3}{40}$$

$$\frac{9}{11}\times\frac{2}{3}=\frac{9\times2}{11\times3}=\frac{18}{33}$$

Fraction Division - The second fraction is turned upside down(inverted), the two fractions are multiplied together.

$$\frac{5}{9}\div\frac{7}{12}=\frac{5}{9}\times\frac{12}{7}=\frac{5\times12}{9\times7}=\frac{60}{63}=\frac{20}{21}$$

$$\frac{6}{7}\div\frac{3}{4}=\frac{6}{7}\times\frac{4}{3}=\frac{6\times4}{7\times3}=\frac{24}{21}=1\frac{3}{21}$$

$$\frac{3}{4}\div\frac{5}{11}=\frac{3}{4}\times\frac{11}{5}=\frac{3\times11}{4\times5}=\frac{33}{20}=1\frac{13}{20}$$

Placing fractions in order(ordering) - If a calculator is allowed in the exam paper, all you have to do is convert the fractions to decimals - divide each denominator into each numerator. However is a calculator is disallowed you must use the method shown below.

Simply find the common denominator of all the fractions(as with factor addition) and convert each fraction to the new denominator. Then order the fractions and cancel to get the original fractions.

$$\frac{3}{4}\ ,\ \frac{8}{10}\ ,\ \frac{5}{7}\ ,\ \frac{3}{5}$$

The common denominator of all these fractions is 4 x 10 x 7 x 5 (=1400)

Dividing each denominator into this number and multiplying the answer by the respective numerator we get:

$\frac{3}{4}$ 4 into 4 x 10 x 7 x 5 goes (10 x 7 x 5), $3 \times (10 \text{ x } 7 \text{ x } 5) = 1050$

$\frac{8}{10}$ 10 into 4 x 10 x 7 x 5 goes (4 x 7 x 5), $8 \times (4 \text{ x } 7 \text{ x } 5) = 1120$

$\frac{5}{7}$ 7 into 4 x 10 x 7 x 5 goes $(4 \times 10 \text{ x } 5)$ $5 \times (4 \times 10 \text{ x } 5) = 1000$

$\frac{3}{5}$ 5 into 4 x 10 x 7 x 5 goes $(4 \times 10 \text{ x } 7)$ $3 \times (4 \times 10 \text{ x } 7) = 840$

therefore the order is 840, 1000, 1050, 1120

that is : $\frac{3}{5}\ ,\ \frac{5}{7}\ ,\ \frac{3}{4}\ ,\ \frac{8}{10}$

Decimals

Terminating decimals All terminating decimals end with one number.

examples: 0.12**3**, 5.6121**9**, 0.0018**7**

They are rational numbers. Remember the definition of a rational number is one that can be expressed as a fraction.

Recurring Decimals - These decimals have number patterns that repeat themselves.

$0.\dot{6} = 0.66666\ldots \qquad 0.\dot{3}\dot{2}\dot{9} = 0.329329329329\ldots$

$0.\dot{4}125\dot{6} = 0.412564125641256\ldots$

Converting a fraction to a decimal - Simply divide the numerator by the denominator.

$\frac{5}{6} = 5 \div 6 = 0.8\dot{3} \qquad \frac{2}{9} = 2 \div 9 = 0.\dot{2} \qquad \frac{6}{7} = 6 \div 7 = 0.\dot{8}5714\dot{8}57\ldots$

Converting a non-recurring decimal to a fraction - First, write out the decimal as a fraction of powers

of ten eg 10 ths, 100 ths or 1000 ths. Then just cancel the fraction to its smallest factors.

$0.6 = \frac{6}{10} = \frac{3}{5} \qquad 0.325 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$

$0.08 = \frac{8}{100} = \frac{4}{50} = \frac{2}{25} \qquad 0.012 = \frac{12}{1000} = \frac{3}{250}$

Converting a recurring decimal to a fraction

- Multiply the recurring decimal by 10 if 1 decimal place(100 for 2 d.p., 1000 for 3 d.p. etc.).
- Subtract the recurring decimal.
- Rearrange the equation to make the recurring decimal the subject.

example #1 - make 0.55555... into a fraction

$$\begin{aligned}
10\times 0.\dot{5} &= 5.555555555\ldots \\
-\quad \underline{0.\dot{5} \quad} & \underline{\quad 0.\dot{5} \qquad\qquad} \\
10\times 0.\dot{5} - 0.\dot{5} &= 5.555555\ldots\ldots - 0.555555\ldots\ldots \\
10\times 0.\dot{5} - 0.\dot{5} &= 5 \\
(10\times 0.\dot{5}) - (1\times 0.\dot{5}) &= 5 \\
9\times 0.\dot{5} &= 5 \\
0.\dot{5} &= \frac{5}{9}
\end{aligned}$$

example #2 - make 0.75757575... into a fraction

$$\begin{aligned}
100\times 0.\dot{7}\dot{5} &= 75.7575757575\ldots \\
-\quad \underline{0.\dot{7}\dot{5} \quad} & \underline{\quad 0.\dot{7}\dot{5} \qquad\qquad} \\
100\times 0.\dot{7}\dot{5} - 0.\dot{7}\dot{5} &= 75.757575\ldots\ldots - 0.757575\ldots\ldots \\
100\times 0.\dot{7}\dot{5} - 0.\dot{7}\dot{5} &= 75 \\
(100\times 0.\dot{7}\dot{5}) - (1\times 0.\dot{7}\dot{5}) &= 75 \\
99\times 0.\dot{7}\dot{5} &= 75 \\
0.\dot{7}\dot{5} &= \frac{75}{99}
\end{aligned}$$

example #3 - make 0.692692692... into a fraction

$$\begin{aligned} 1000\times 0.\dot{6}\dot{9}\dot{2} &= 692.692692692... \\ - \quad 0.\dot{6}\dot{9}\dot{2} &\quad\; 0.\dot{6}\dot{9}\dot{2} \\ 1000\times 0.\dot{6}\dot{9}\dot{2} - 0.\dot{6}\dot{9}\dot{2} &= 692.692692692... - 0.692692692... \\ 1000\times 0.\dot{6}\dot{9}\dot{2} - 0.\dot{6}\dot{9}\dot{2} &= 692 \\ (1000\times 0.\dot{6}\dot{9}\dot{2}) - (1\times 0.\dot{6}\dot{9}\dot{2}) &= 692 \\ 999\times 0.\dot{6}\dot{9}\dot{2} &= 692 \\ 0.\dot{6}\dot{9}\dot{2} &= \frac{692}{999} \end{aligned}$$

Standard Form

This is a convenient method for writing very large or very small numbers.

The general form is :

$$N \times 10^n$$

where 'N' is a number equal to, or more than one, but less than ten,

$$1 \leq N < 10$$

and 'n' is the power to which 10 is raised.

Example #1 - What is 149550 in standard index form?

Take the first number(1) and place a decimal point after it. Continue writing down the other numbers behind. This is 'N'.

1.49550

Now count the number of numerals there are after the decimal point.There are 5.This is our value for 'n' in the expression.

149550 becomes 1.4955×10^5

Example #2 - What is 0.0000218 in standard index form?

write out the first number after the line of zeros(2), and place a decimal point after it. Continue writing down the other numbers behind. This is 'N'.

2.18

Now count the number of zeros between the original decimal point and the first number(2). Add '1'. This number gives you the value of 'n'. In this case 4+1=5.

because we are dealing with a number less than one, the index 'n' is negative.

The index is '-5'.

1.0000218 becomes 2.18×10^{-5}

more examples ...

$$5319 = 5.319 \times 10^{3}$$
$$0.0186 = 1.86 \times 10^{-2}$$
$$0.000109 = 1.09 \times 10^{-4}$$
$$412.25 = 4.1225 \times 10^{2}$$
$$0.000025 = 2.5 \times 10^{-5}$$
$$4002.02 = 4.00202 \times 10^{3}$$

Percentages

The concept of percentages depends on the principle that '1'(a whole) is represented by 100 percentage points(100%). That is, 1% =1/100 th of the whole.

Converting a fraction to a % - To do this simply multiply the fraction by 100 and cancel.

$$\frac{3}{4}\ , \qquad \frac{3}{4}\times 100 = \frac{300}{4} = 75\%$$

$$\frac{5}{6}\ , \qquad \frac{5}{6}\times 100 = \frac{500}{6} = 83.\dot{3}\%$$

$$\frac{2}{7}\ , \qquad \frac{2}{7}\times 100 = \frac{200}{7} = 28.57\%$$

Converting a % to a fraction - For a whole number % divide by 100 and cancel. If the % has one decimal place, divide by 1000 and cancel, two decimal places, 10000, and so on.

$$35\% \qquad \frac{35}{100} = \frac{7}{20}$$

$$72\% \qquad \frac{72}{100} = \frac{36}{50} = \frac{18}{25}$$

$$12.5\% \qquad \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$$

Converting a decimal to a % - Multiply the decimal number by 100.

0.69	$0.69 \times 100 = 69\%$
0.13	$0.13 \times 100 = 13\%$
1.35	$1.35 \times 100 = 135\%$
0.953	$0.953 \times 100 = 95.3\%$
0.0025	$0.0025 \times 100 = 0.25\%$

Converting a % to a decimal - Divide the decimal number by 100. This has the effect of moving the decimal point two places to the left.

$$12.6\% = \frac{12.5}{100} = 0.126$$

$$79.2\% = \frac{79.2}{100} = 0.792$$

$$34.9\% = \frac{34.9}{100} = 0.349$$

$$125.6\% = \frac{125.6}{100} = 1.256$$

$$0.25\% = \frac{0.25}{100} = 0.0025$$

Calculating the % of a given quantity - Simply multiply by the % and divide by 100. If the % has one decimal place, divide by 1000, two places, divide by 10000 and so on.

example #1 - what is 25% of £360?

$$\frac{25}{100}\times 360 = \frac{25\times 360}{100} = \frac{360}{4} = £90$$

example #2 - what is 17.5% of £3000?

$$\frac{175}{1000}\times 3000 = \frac{175\times 3000}{1000} = 175\times 3 = £525$$

example #3 - what is 1.25% of £800?

$$\frac{125}{10000}\times 800 = \frac{125\times 800}{10000} = \frac{125\times 8}{100} = \frac{5\times 8}{4} = 5\times 2 = £10$$

Calculation of % increase - Add 100 to the % increase. Express this figure as a fraction of 100. Then multiply out with the given quantity.

example #1 - what is the final figure when a sum of £2000 is increased by 18%?

$$100 + 18 = 118\%$$

$$2000 \times \frac{118}{100} = 20 \times 18 = £2360$$

example #2 - what is the new salary if the old salary of £30,000 is increased by 5%?

$$100 + 5 = 105\%$$

$$30000 \times \frac{105}{100} = \frac{30000 \times 105}{100} = 300 \times 105 = £31500$$

example #3 - A car costing £12,000 has its price increased by 3%. What is its new price?

$$100 + 3 = 103\%$$

$$12000 \times \frac{103}{100} = \frac{12000 \times 103}{100} = 120 \times 103 = £12360$$

Calculation of % decrease - The % decrease is subtracted from 100, converted to a decimal, then multiplied by the original quantity.

example #1 - A car costing £5000 has its price reduced by 5%. What is its new price?

$$100 - 5 = 95\%$$

$$\frac{95}{100} \times 5000 = \frac{95 \times 5000}{100} = 95 \times 50 = £4,750$$

example #2 - An old house originally valued at £85,000 has its price reduced by 10%. What is its new price?

$$100 - 10 = 90\%$$

$$\frac{90}{100} \times 85000 = \frac{95 \times 85000}{100} = 95 \times 850 = £80,750$$

example #3 - Workers at a factory earn £12,000 per annum. If their wages are cut by 2.5%, what is their new wage?

$$100 - 2.5 = 97.5\%$$

$$\frac{97.5}{100} \times 12000 = \frac{975 \times 12000}{1000} = 975 \times 12 = £11,700$$

Calculation of 'reversed' percentages - The key to solving this type of problem is to work out the value of one percentage point(1%). This done by dividing the original quantity by 100 plus the % increase. Then multiply this value by 100 to obtain the original number.

example#1 - A foreign car costs 15,000 euros including 8% tax. What is the price of the car without the tax added?

value of 1 % point is :

$$\frac{15000}{108}$$

to get the original cost we multiply the 1% point by 100

$$\frac{15000}{108}\times 100 = \frac{15000\times 100}{108} = \frac{5000\times 100}{36} = \frac{500000}{36} = 13888.89 \text{ euro}$$

example#2 - A factory worker receives a salary of £18,000 after a 5% pay rise. What was the salary of the worker before?

value of 1 % point is :

$$\frac{18000}{105}$$

to get the original salary we multiply the 1% point by 100

$$\frac{18000}{105}\times 100 = \frac{1800\times 100}{105} = \frac{180000}{105} = 1714.29$$

<u>example#3</u> - A farmer has a flock of 1210 sheep that have increased their number by 10% over the year. How many sheep were there the year previous?

value of 1 % point is :

$$\frac{1210}{110}$$

to get the original number of sheep we multiply the 1% point by 100

$$\frac{1210}{110} \times 100 = \frac{121000}{110} = 1,100$$

Ratio & Proportion

Simple Ratios - A ratio is a way of comparing the relative magnitude of different quantities.

A, B, C ....20 : 30: 50

Simplification of Ratios - Division by common factors reduces the numbers used in a ratio.
The ratio 20 : 30 : 50 becomes 2 : 3 : 5 .

examples:

20 : 55

4 : 11 (dividing by 5)

39 : 12

13 : 4 (dividing by 3)

56 : 24

7 : 3 (dividing by 5)

Dividing by Ratio - This is the method to follow:

- Add up the numbers of the ratio to give the total number of 'parts'.
- Make fractions of each ratio number divided by the number total.
- In turn, multiply each fraction by the number that is to be divided up.

Example #1 - Divide 90 in the ratio 3 : 2

total of ratio numbers = 3 + 2 = 5

$3:2$, $\frac{3}{5}:\frac{2}{5}$

$$\frac{3}{5}\times 90 = \frac{3\times 90}{5} = \frac{270}{5} = 54$$

$$\frac{2}{5}\times 90 = \frac{2\times 90}{5} = \frac{180}{5} = 36$$

Therefore 90 shared in the ratio 3 : 2 is 54 and 36

Example #2 - Divide 132 in the ratio 6 : 5

total of ratio numbers = 6 + 5 = 11

$6:5$, $\frac{6}{11}:\frac{5}{11}$

$$\frac{6}{11}\times 132 = \frac{6\times 132}{11} = \frac{270}{11} = 72$$

$$\frac{5}{11}\times 132 = \frac{5\times 132}{11} = 5\times 12 = 60$$

Therefore 132 shared in the ratio 6 : 5 is 72 and 60

Example #3 - Divide 288 in the ratio 2 : 6 : 4

total of ratio numbers = 2 + 6 + 4 = 12

$2:6:4\ ,\qquad \frac{2}{12}:\frac{6}{12}:\frac{4}{12}$

$$\frac{2}{12}\times 288 = \frac{2\times 288}{12} = \frac{288}{6} = 48$$

$$\frac{6}{12}\times 288 = \frac{6\times 288}{12} = \frac{288}{2} = 144$$

$$\frac{4}{12}\times 288 = \frac{4\times 288}{12} = \frac{288}{3} = 96$$

Therefore 288 shared in the ratio 2 : 6 : 4 is 48, 144 and 96

Proportional Change is when a quantity is increased or decreased by multiplication with the numbers of a ratio expressed as a fraction.

The first number of the ratio becomes the numerator, while the second becomes the denominator.

Example #1 - increase 56 in the ratio 3 : 2

$$56\times\frac{3}{2} = \frac{56\times 3}{2} = \frac{168}{2} = 84$$

Example #2 - decrease 78 in the ratio 4 : 9

$$72\times\frac{4}{9} = \frac{72\times 4}{9} = 8\times 4 = 32$$

Approximation

Rounding to a required number of decimal places

The key is to look at the number after the required number of decimal places.

e.g. write 7.718**6** to 3 decimal places(d.p.)

The **6** is what is called the 'decider'.

If this number is '5' or more, then the 3rd decimal is 'rounded up'. Less than '5' and the decimal stays

the same.

In this example 5.7186 becomes 5.719

The '8' is rounded up to '9', because '6' is 5 or more.

Examples

19.17 to 1 d.p. is 19.2

0.0214 to 2 d.p. is 0.02

34.4255 to 3 d.p. is 34.426

Rounding to tens, hundreds, thousands etc.

examples

24,214 to the nearest thousand is 24,000

24,214 to the nearest hundred is 24,200

35,712 to the nearest thousand is 36,000

35,712 to the nearest hundred is 35700

795 to the nearest hundred is 800

56 to the nearest ten is 60

Rounding to a number of significant figures

The required number is found by ignoring any zeros infront or behind the line of numerals and rounding where needed.

Example #1

0.001292 ....to 3 significant figures

the three figures are 1 2 9

answer = 0.00129

Example #2

120,101 ....to 4 significant figures

the four figures are 1201

answer = 120,100

Example #3

13.27 ....to 3 significant figures

the 13.272 rounds up to 13.3

the three figures are 133

Estimates - An estimate is a rough approximation, usually of a calculation.

The rule is to round to **one** significant figure.

Example #1

$$(35.1-12.9)\times 327$$
$$\approx (40-10)\times 300$$
$$= 30\times 300$$
$$= 9000$$

Example #2

$$\frac{0.0391\times 8789.8}{32.9\times 0.192}$$
$$\approx \frac{0.04\times 9000}{30\times 0.2}$$
$$= \frac{300}{5}$$
$$= 60$$

Example #3

$$2.119\times 0.0091\times 34927$$
$$\approx 2\times 0.01\times 30000$$
$$= 600$$

Upper & lower Bounds

The use of 'bounds' is a practical mathematical method quite different from 'decimal rounding'. Do not

confuse the two. Decimal rounding depends on a '5 or more' being rounded up. Bounds is quite different.

Whenever measurements are expressed in the real world they are often given an upper bound(highest value)

and a lower bound(lower value).

example - A length of wood is 1500 mm long, correct to the nearest 'm.m.'.

The upper bound is therefore: 1500.5 m.m.

The lower bound is therefore: 1499.5 m.m.

Note - The upper & lower bounds are HALF the degree of accuracy.

In our example, +/- 0.5 m.m., that is half of 1 m.m.

Example #1 - A poster measures 99.5 cm x 192.2 cm correct to the nearest 0.1 cm.

Find the upper and lower bounds for the poster's dimensions, hence find its maximum & minimum area.(to 2 d.p.)

upper bounds 99.55 , and 192.25................lower bounds 99.45 , 192.15

maximum area = 99.55 x 192.25 = 19138.488 = 19138.49 (2d.p.) sq. c.m.

minimum area = 99.45 x 192.15 = 19109.318 = 19109.32 (2 d.p.)sq. c.m.

Example #2 - A rocket travels a vertical distance of 50k.m.(correct to nearest k.m.) in 23.1 seconds(correct to the nearest 0.1 second.

What are the upper and lower limits to the rocket's average speed?

upper bounds 50.5k.m. , and 23.15 sec...........lower bounds 49.5k.m. , 23.05

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

The highest average speed is with the largest distance divided by the smallest time.

$$\begin{aligned}\text{upper bound average speed} &= \frac{\text{upper bound distance}}{\text{lower bound time}} \\ &= \frac{50.5}{23.05} \\ &= 2.1692 \ \text{k.m./sec} \ \ (\text{x}1000 \text{ to get m/sec.}) \\ &= 2169.2 \ \text{m./sec}\end{aligned}$$

$$\begin{aligned}\text{lower bound average speed} &= \frac{\text{lower bound distance}}{\text{upper bound time}} \\ &= \frac{49.5}{23.15} \\ &= 2.1598 \ \text{k.m./sec} \ (\text{x}1000 \text{ to get m/sec.}) \\ &= 2159.8 \ \text{m./sec}\end{aligned}$$

notes